10.4 Defuzzification Methods

Defuzzification is the process of conversion of a fuzzy quantity into a precise quantity. The output of a fuzzy process may be union of two or more fuzzy membership functions defined on the universe of discourse of the output variable.

Consider a fuzzy output comprising two parts: the first part, $\underset{\sim}{C}_1$, a triangular membership shape [as shown in Figure 10-3(A)], the second part, $\underset{\sim}{C}_2$, a trapezoidal shape [as shown in Figure 10-3(B)]. The union of these two membership functions, i.e., $\underset{\sim}{C} = \underset{\sim}{C}_1 \cup \underset{\sim}{C}_2$ involves the max-operator, which is going to be the outer envelope of the two shapes shown in Figures 10-3(A) and (B); the final shape of $\underset{\sim}{C}$ is shown in Figure 10-3(C).

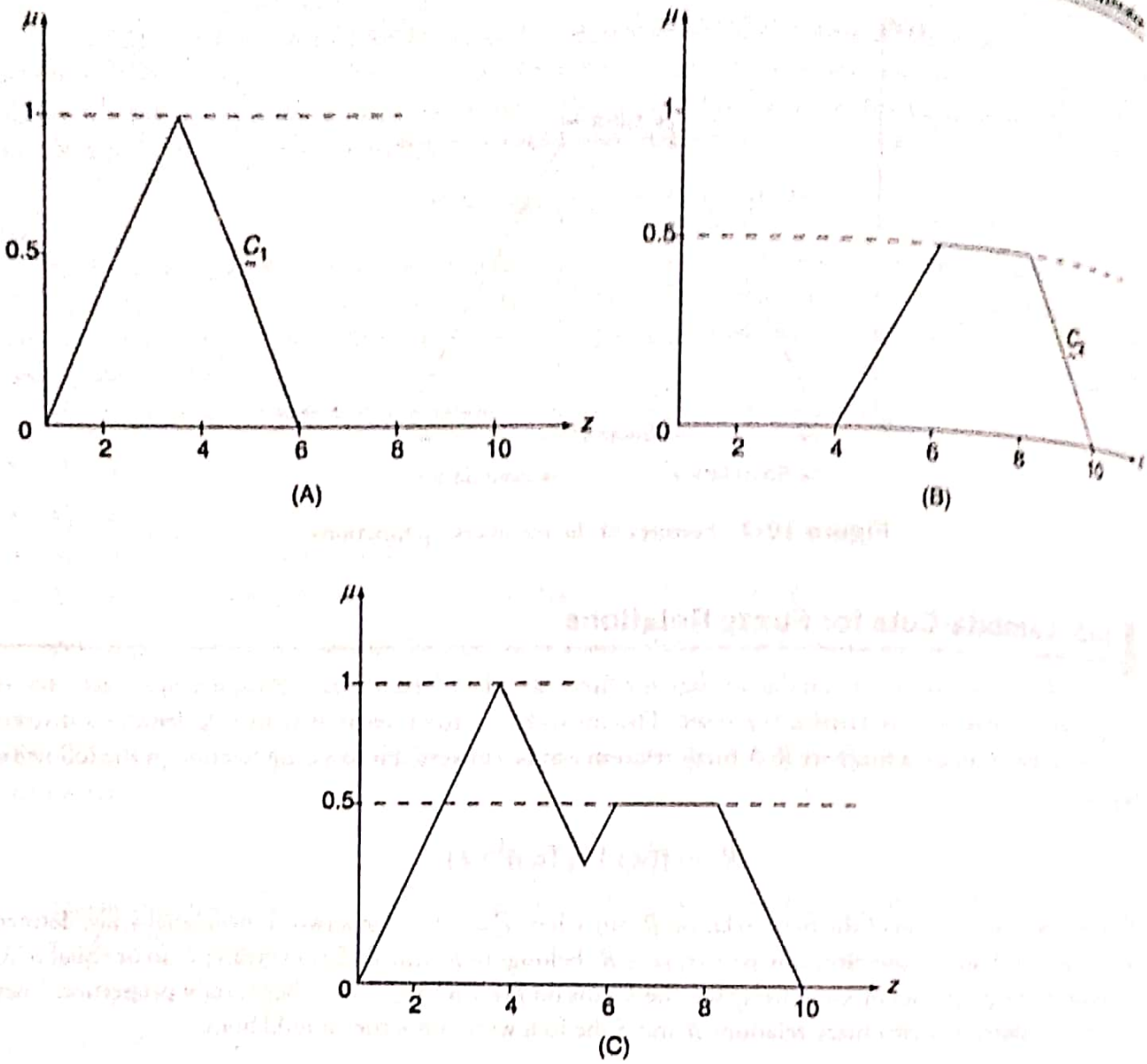

Figure 10-3 (A) First part of fuzzy output, (B) second part of fuzzy output, (C) union of parts (A) and (B).

A fuzzy output process may involve many output parts, and the membership function representing each part of the output can have any shape. The membership function of the fuzzy output need not always be normal. In general, we have

$$C_n = \bigcup_{i=1}^{n} C_i = C$$

Defuzzification methods include the following:

1. Max-membership principle.
2. Centroid method.
3. Weighted average method.
4. Mean-max membership.

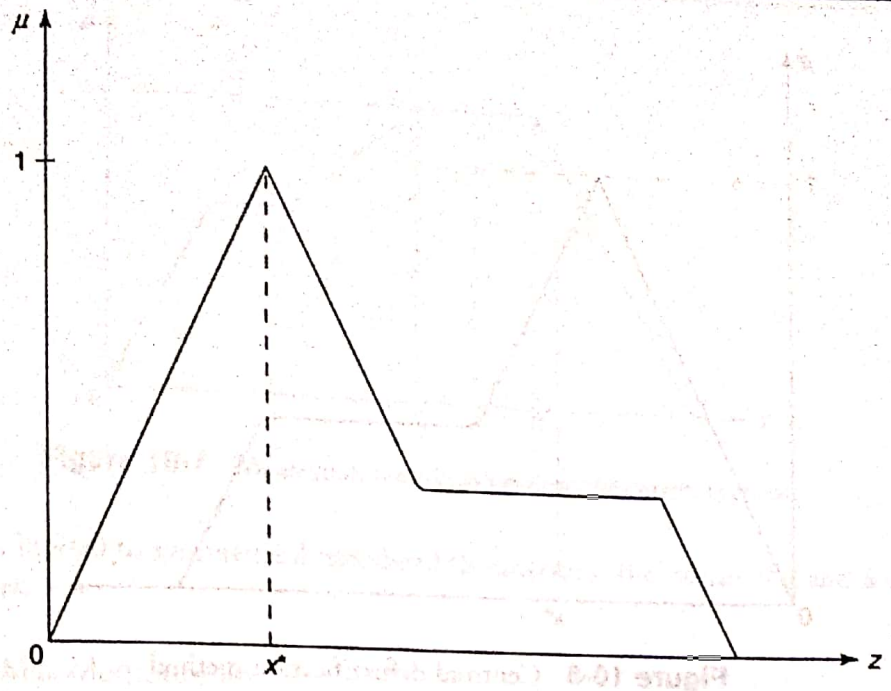

Figure 10-4 Max-membership defuzzification method.

5. Center of sums.
6. Center of largest area.
7. First of maxima, last of maxima.

Now we discuss the methods listed above.

10.4.1 Max-Membership Principle

This method is also known as height method and is limited to peak output functions. This method is given by the algebraic expression

$$\mu_C(x^*) \geq \mu_C(x) \text{ for all } x \in X$$

The method is illustrated in Figure 10-4.

10.4.2 Centroid Method

This method is also known as center of mass, center of area or center of gravity method. It is the most commonly used defuzzification method. The defuzzified output x^* is defined as

$$x^* = \frac{\int \mu_C(x) \cdot x dx}{\int \mu_C(x) dx}$$

where the symbol $\int$ denotes an algebraic integration. This method is illustrated in Figure 10-5.

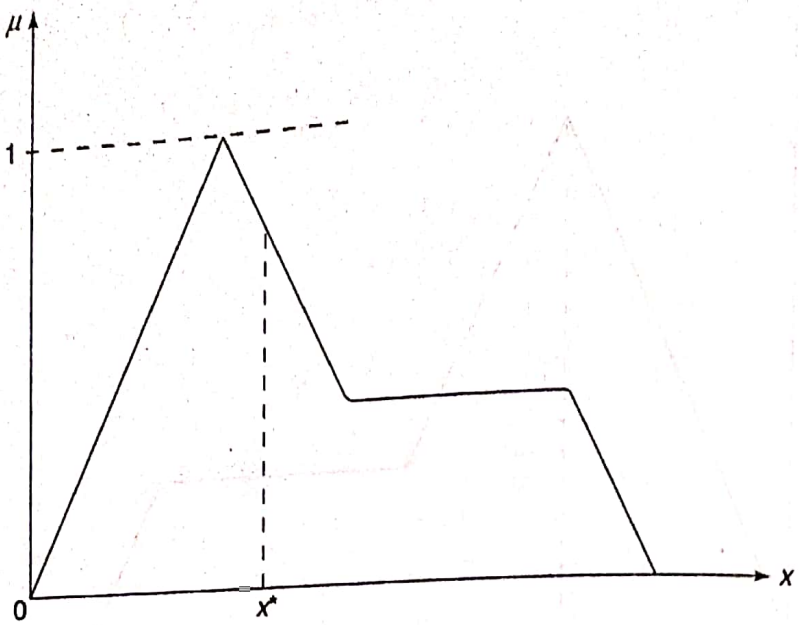

Figure 10-5 Centroid defuzzification method.

10.4.3 Weighted Average Method

This method is valid for symmetrical output membership functions only. Each membership function is weighted by its maximum membership value. The output in this case is given by

$$x^* = \frac{\sum \mu_C(\bar{x}_i) \cdot \bar{x}_i}{\sum \mu_C(\bar{x}_i)}$$

where $\sum$ denotes algebraic sum and $\bar{x}_i$ is the maximum of the ith membership function. The method is illustrated in Figure 10-6, where two fuzzy sets are considered. From Figure 10-6, we notice that the defuzzified output is given by

$$x^* = \frac{0.5a + 0.8b}{0.5 + 0.8}$$

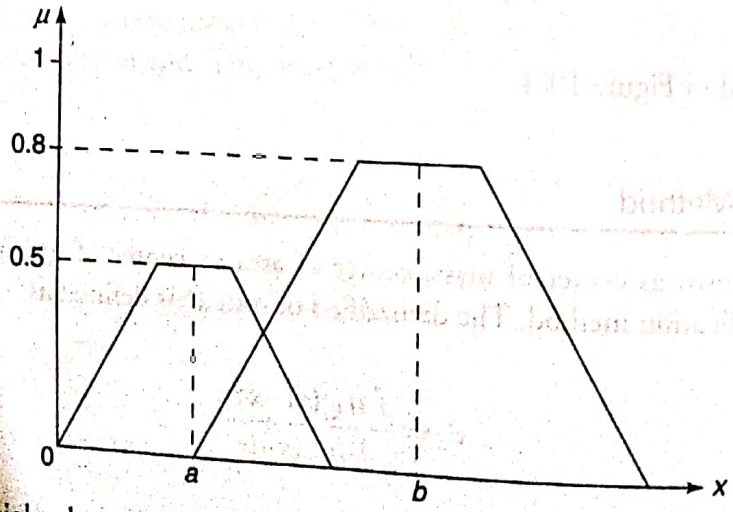

Figure 10-6 Weighted average defuzzification method (two symmetrical membership functions).

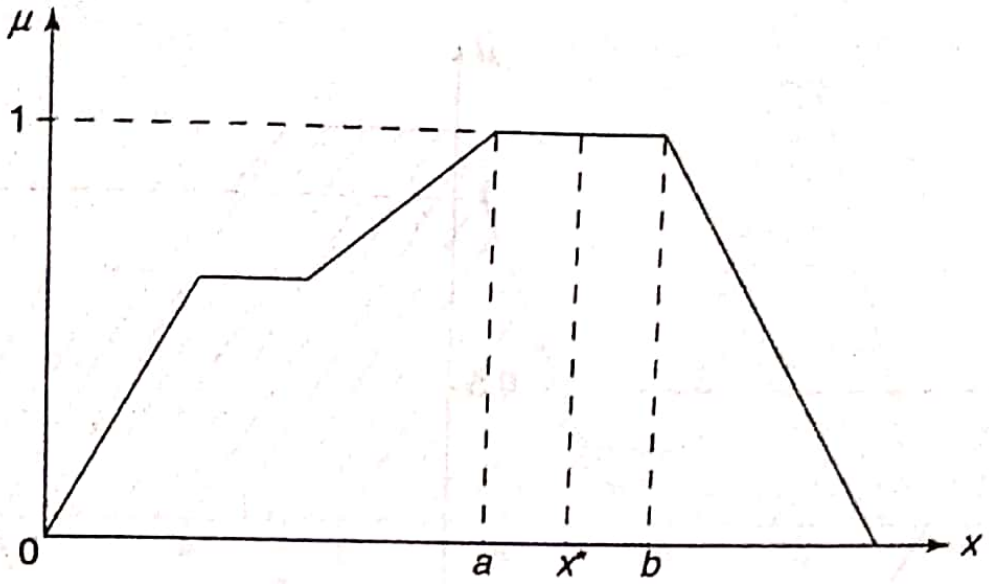

Figure 10-7 Mean-max membership defuzzification method.

As this method is limited to symmetrical membership functions, the values of *a* and *b* are the means of their respective shapes.